Evaluating Fracture Parameters from Phase Field Simulations

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Abstract. Phase field models (PFMs) are developed from Griffith's energy balance formulation, which find an alternative route to numerically model the fracture process. The beautiful part of using phase field modelling is that it does not require additional crack initiation and propagation criterion. The ability of the phase field models to predict crack paths has been demonstrated in various literature for different classes of materials. But, the fracture parameter evaluation has not been carried out in any of these studies. This work addresses this using photoelasticity for a simple SEN specimen.

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Introduction

Griffith's energy balance has been the key foundation for understanding the fracture in materials. Numerical techniques methods like Finite Element Method (FEM), Finite Volume Method (FVM), Boundary Element Method (BEM) can be used implement phase field. Phase field models (PFMs) use Griffith's principles and offer advantages over the traditional numerical methods like XFEM in modelling fracture. Unlike other simulation methods, the damage is captured by introducing an additional variable called *phase field variable* that distinguishes the between the phases, viz. damaged and undamaged states. PFMs have the ability to simulate the fracture process in complex scenarios without additional criteria for crack initiation and propagation. Despite these advancements, ensuring the accuracy of the numerical simulations remains a concern [1, 2]. In this situation, experimental validation becomes a crucial aspect and evaluation of the stress intensity factors (SIFs) are not reported in the PF literature.
Photoelasticity has been a useful tool in devising many concepts in the fracture literature and is still emerging as a useful tool for validating PF models. This work presents a framework for post-processing the numerical isochromatics for estimating the SIF from PF models. Integrating this technique into PF simulations helps in validating fracture simulations[1].

Background

At point in the domain (Ω) considering the hybrid formulation by Ambati *et al.*[3], the displacement and phase field variable can be computed by solving two coupled PDEs given by:

$$\nabla \cdot \boldsymbol{\sigma} = 0 \text{ in } \Omega$$

$$-G_c \ell_o \nabla^2 d + \left[\frac{G_c}{\ell_o} + 2H^+ \right] d = 2H^+ \text{in } \Omega$$

where the $\boldsymbol{\sigma} = g(d)\frac{\partial\psi(\boldsymbol{\varepsilon})}{\partial\boldsymbol{\varepsilon}}$: Cauchy stress tensor, $\boldsymbol{\varepsilon} = \mathrm{sym}(\nabla u)$: small strain tensor, $g(d) = ((1-d)^2 + k)$: degradation function and $\partial\psi(\boldsymbol{\varepsilon}) = \frac{1}{2}\lambda(\mathrm{tr}\,(\boldsymbol{\varepsilon}))^2 + \mu\mathrm{tr}\,(\boldsymbol{\varepsilon}^2)$ is the elastic energy with Lame's constants, λ and μ. The size of the regularized surface is governed by length scale parameter ℓ_o. The history variable H^+ considers the positive energy part ψ^+ to avoid the interpenetration of crack surfaces. These equations with the respective boundary conditions can be weakened with the help of the standard Bubnov-Galekerin process, and a staggered method is used to solve Partial Differential Equations (PDE's) [4] using a open-source finite element package FeniCS [5].

Photoelasticity is a whole-field technique which provides the difference in the principal stresses, known as isochromatics. For plotting the fringes as in photoelasticity, one has to extract the principal stress difference at that point in FE analysis and assign a suitable fringe order based on equation (4). This has been elucidated in Ref. [1]. Pixel-wise fringe order data is required to determine the SIF from the isochromatic plot. Since a fine mesh is used near the crack tip in PFMs, whole field fringe order data can be developed using the filters in ParaView[6]. The data collected (x, y, N) or (r, θ, N) is used for over-deterministic non-linear least square analysis based on the corrected Atluri and Kobayashi equations for mixed-mode conditions. Based on the satisfactory theoretical reconstruction of the fringe field and convergence criteria, fracture parameters can be determined for different class of problems [7].

Experiments with SEN specimen

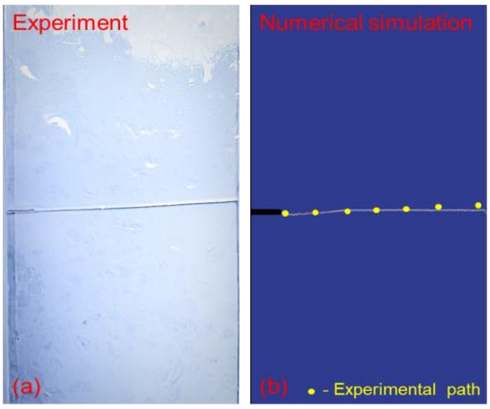

Fig. 1 (a) Crack path in experiment (b) crack path from phase field model

A rectangular specimen of 100 mm × 40 mm × 2 mm made from PMMA with an edge crack of 5 mm is used to visualize the stress fields. Numerical simulations were carried out using the phase field. The crack path was similar to the experiments (Fig. 1) and the numerical isochromatics were plotted using the F_σ = 140 N/mm/Fringe. Due to the high F_σ value, the isochromatic patterns are also not visible in the numerical simulations as well as experiments Fig. 2(a and b).

One alternative to calculate SIF from the numerical simulation is to plot the isochromatics using a pseudo F_σ for getting required number of the fringe orders. This is due to fact that the F_σ does not affect the SIF value during least square analysis. A pseudo F_σ = 5 N/mm/Fringe was used to generate the isochromatics as shown in Fig. 2(c). The complete process outlined in Ref. [7] can be used to generate the fringe order data file from ParaView, which can be used in PSIF[8] (an in-house software to determine the SIF values using the least squares method). The SIF value from Tada *et al.* [9] (accuracy better than 0.5% for an *a*/*w* ratio) is found to be 1.417 MPa√m for a load of 9.1 MPa at crack initiation. The SIF value using the least squares analysis using PSIF was 1.468 MPa√m. The PSIF required 7 parameters to reconstruct the fringe field, ensuring the convergence below 0.05.

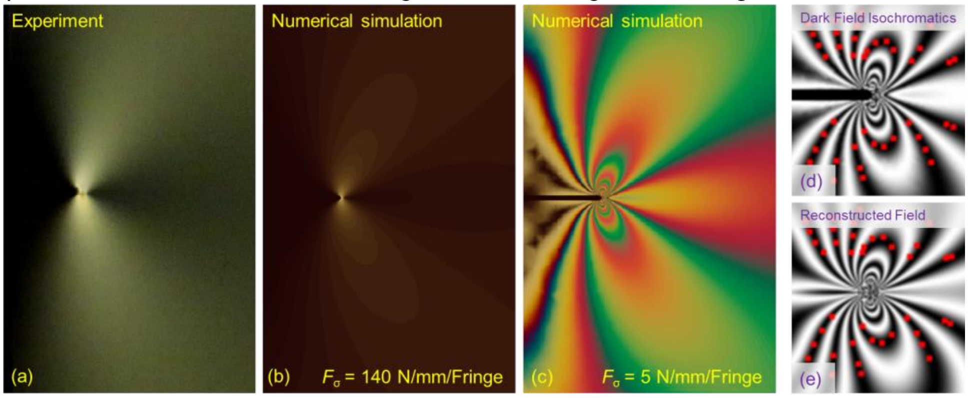

Fig. 2 (a) Experimentally observed isochromatics at crack initiation (b) Numerically simulated isochromatics using F_σ = 140 N/mm/Fringe (c) Numerically simulated isochromatics using pseudo F_σ = 5 N/mm/Fringe (d) Dark field isochromatics with data points collected (red points) (e) Reconstructed fringe field using PSIF®.

Conclusion

Generally, phase field methods are verified using crack path and peak load values. An alternative methodology using photoelasticity has been explained in this work, which compares the stress fields in the experiments and numerical simulations. In the cases where the material shows less response in the photoelasticity, one can numerically simulate the problem and generate isochromatics using an arbitrary pseudo F_σ value. Using this, SIF evaluation can be conducted using the PSIF software. This method can also be extended to materials that are not optically responsive to photoelasticity.

Literature

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