Equivalence of Multiparameter Stress Field Equations for a Bimaterial Interfacial Crack

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Abstract. This study presents a comparison of two multiparameter stress field equations for a bimaterial interface crack: the Deng's equation and the Chen's equation. The Deng's equation was for a crack along the interface, while the Chen's equation was derived for a crack at an arbitrary angle to and terminating at the interface. The comparison effected in this study shows that both equations can be simplified to the form of that given by Atluri and Kobayashi for the homogeneous case, and a relationship between the coefficients of the stress field equations is also established.

Possible Sessions

1. Fatigue and fracture 2. Adhesive and welded joints

Introduction

Bimaterial interfacial cracks are of great importance in engineering and material science due to their common occurrence across various applications such as welded structures, electronic packages, bimetal thermostats, composite structures, ceramic coatings, geological investigations. Understanding the effects of these cracks on the boundary between different materials is crucial for designing reliable structures and components, which in turn improves the material performance and ensures safety and longevity of various structural components. The knowledge about the stress field around the cracks in such a case is essential for fracture mechanics, stress analysis, and material selection. Though the problem of a crack in a homogeneous material is well understood and considerable progress has been made in comprehending the stress field nearer to the crack tip, the stress distribution would become extremely different and complex when the crack is present in a bimaterial interface. Various researchers have approached this problem and have proposed different stress field equations [1–4]. All these solutions were intricate and not straightforward for experimentalists to utilize.

Multiparameter solutions have proven invaluable to experimentalists for determining fracture parameters, including stress intensity factor and *T*-stress from photoelastic fringe patterns, especially in the context of homogeneous materials. However, for the case of bimaterials, the initial solutions available were confined to only singular terms. Ravichandran and Ramesh proposed a multiparameter stress field equation that can be directly used by the experimentalists for evaluating the stress field parameters [5] by simplifying the Deng's equation [6]. This solution was restricted only to the case of cracks that lie along the interface. Researchers have effectively utilized this equation to determine the fracture parameters through the application of photoelasticity, demonstrating the equation's efficacy [7,8]. Chen derived a generic multiparameter stress field equation for a crack located at an arbitrary angle to and terminating at the interface of the bimaterial using the eigen function expansion method [9]. The derived expressions have been used by him in some of the fracture studies regarding the bimaterial cracks [10,11]. However, the elegance of the Chen's equation was not appreciated by the researchers for a long period of time till Vivekanandan and Ramesh used the Chen's equations to study the effect of crack inclination angle and mode mixity on the stress intensity factor of bimaterial interface crack which is located at an arbitrary angle to the interface [12,13].

The Deng's and Chen's equations, in essence, stand as the two readily applicable multiparameter stress field equations in the literature where the researchers can conveniently use them for analysing the fracture of bimaterials using the experimental techniques like photoelasticity. This study investigates how these equations are related to the Atluri and Kobayashi equation [14] for the homogeneous case. In the case of cracks in a homogeneous material, through suitable parameter substitutions and subsequent simplifications, this study has brought out that both equations converge to the Atluri and Kobayashi equation. Further, a relationship between the coefficients in the bimaterial stress field equations and Atluri and Kobayashi equation is also elucidated.

Comparison of the Bimaterial Stress Field Equations (Deng's and Chen's Equations)

To compare the equations provided by Deng and Chen, the Atluri and Kobayashi multiparameter equation is taken as the base as it is the most elegant form of stress field expression for the case of a crack present in a homogeneous material. For the homogenous case, after substituting the parameters in the expressions given by Chen and Deng, it is observed that both equations converge to the similar form of the equation

given by Atluri and Kobayashi and it is observed that a relationship exists between the coefficients in the respective expressions as given in Eq. (1).

$$A_{Ij} = \frac{1}{j\sqrt{2\pi}} K_{I(j-1)} = \begin{cases} b_{r((j+1)/2)} & \text{if } j \text{ is odd} \\ d_{r(j/2)} & \text{if } j \text{ is even} \end{cases}$$

$$A_{IIj} = \frac{-1}{j\sqrt{2\pi}} K_{II(j-1)} = \begin{cases} b_{i((j+1)/2)} & \text{if } j \text{ is odd} \\ d_{i(j/2)} & \text{if } j \text{ is even} \end{cases} \quad (1)$$

$$j = 1, 2, 3, \ldots n$$

where *A* corresponds to the coefficients in Atluri and Kobayashi equation, *K* corresponds to the coefficients in Deng's equation while *b* and *d* correspond to the coefficients in Chen's equation. To further validate the equivalence of the equations and to establish the relationship between the coefficients, isochromatic fringe patterns (contours of constant principal stress differences obtained from the technique photoelasticity) for the dark field arrangement are plotted using the stress field equations by Atluri and Kobayashi, Deng and Chen. The values of the parameters are selected in such a way that the relationships established in Eq. (1) hold between them. The isochromatics thus plotted are observed to be exactly similar for the three cases and is shown in Fig. 1.

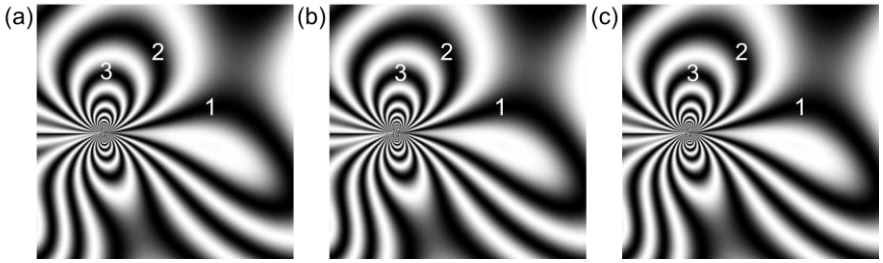

Fig. 1 Isochromatics plotted using (a) Atluri and Kobayashi, (b)Deng's and (c) Chen's equation

Conclusion

The significance of Deng's and Chen's multiparameter stress field equations in the case of a bimaterial interface crack, particularly when employing experimental techniques such as photoelasticity is highlighted. The Deng's equation specifically addresses cracks along the interface, whereas Chen's equation, characterized as the most generic form, provides the stress field expressions for a crack at an arbitrary angle to and terminating at the interface of the bimaterial. Remarkably, for the homogeneous case, both equations can be effectively simplified into a form akin to the Atluri and Kobayashi multi-parameter stress field equation which is demonstrated in this study using isochromatic plots, and a relationship between the coefficients in these stress field equations are also established.

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